UNIVERSITY OF MUMBAI

Syllabus

for S. Y. B. Sc. / S. Y. B. A. Semester III & IV (CBCS)

Program: B. Sc. / B. A.

Course: Mathematics

with effect from the academic year 2021-2022

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EDUCATION IN SERVICE OF THE SOCIETY
DEVKIBA COLLEGE
SMT. DEVKIBA MOHANSINHJI CHAUHAN
COLLEGE OF COMMERCE & SCIENCE

(UNIVERSITY OF MUMBAI)

Syllabus for: S.Y.B.Sc./S.Y.B.A.

Program: B.Sc./B/A.
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SEMESTER III

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SEMESTER IV

Course Code	UNIT	TOPICS	Credits	L/Week
Multivariable Calculus I				
USMT 401, UAMT 401	I	Functions of several variables	2	3
	II	Differentiation of Scalar Fields		
	III	Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields		
Linear Algebra II				
USMT 402 ,UAMT 402	I	Linear transformation, Isomorphism, Matrix associated with L.T.	2	3
	II	Inner product spaces		
	III	Eigen values, eigen vectors, diagonalizable matrix		
Numerical methods (Elective A)				
USMT 403A	I	Solutions of algebraic and transcendental equations	2	3
	II	Interpolation, Curve fitting, Numerical integration		
	III	Solutions of linear system of Equations and eigen value problems		
Statistical methods an their applications(Elective B)				
USMT 403B	I	Descriptive Statistics and random variables	2	3
	II	Probability Distribution and Correlation		
	III	Inferential Statistics		
PRACTICALS				
USMTP04		Practicals based on USMT401, USMT 402 and USMT 403	3	5
UAMTP04		Practicals based on UAMT401, UAMT 402	2	4

Teaching Pattern for Semester III

1. Three lectures per week per course. Each lecture is of 48 minutes duration.
2. One Practical (2L) per week per batch for courses USMT301, USMT 302 combined and one Practical (3L) per week for course USMT303 (the batches tobe formed as prescribed by the University. Each practical session is of 48 minutes duration.)

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Semester-III

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

USMT301/ UAMT301: Calculus III

Unit I. Infinite Series (15 Lectures)

1. Infinite series in $\mathbb{R}$. Definition of convergence and divergence. Basic examples including geometric series. Elementary results such as if $\sum_{n=1}^{\infty} a_n$ is convergent, then $a_n \longrightarrow 0$ but converse not true. Cauchy Criterion. Algebra of convergent series.
2. Tests for convergence: Comparison Test, Limit Comparison Test, Ratio Test (without proof), Root Test (without proof), Abel Test (without proof) and Dirichlet Test (without proof). Examples. The decimal expansion of real numbers. Convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ $(p > 1)$. Divergence of harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$.
3. Alternating series. Leibnitz's Test. Examples. Absolute convergence, absolute convergence implies convergence but not conversely. Conditional Convergence.

Unit II. Riemann Integration (15 Lectures)

1. Idea of approximating the area under a curve by inscribed and circumscribed rectangles. Partitions of an interval. Refinement of a partition. Upper and Lower sums for a bounded real valued function on a closed and bounded interval. Riemann integrability and the Riemann integral.
2. Criterion for Riemann integrability. Characterization of the Riemann integral as the limit of a sum. Examples.

3. Algebra of Riemann integrable functions. Also, basic results such as if $f : [a, b] \longrightarrow \mathbb{R}$ is integrable, then (i) $\displaystyle\int_a^b f(x)\ dx = \int_a^c f(x)\ dx + \int_c^b f(x)\ dx$. (ii) $|f|$ is integrable and $\displaystyle\left|\int_a^b f(x)\ dx\right| \leq \int_a^b |f|(x)\ dx$ (iii) If $f(x) \geq 0$ for all $x \in [a, b]$ then $\displaystyle\int_a^b f(x)\ dx \geq 0$.
4. Riemann integrability of a continuous function, and more generally of a bounded function whose set of discontinuities has only finitely many points. Riemann integrability of monotone functions.

Unit III. Applications of Integrations and Improper Integrals (15 lectures)

1. Area between the two curves. Lengths of plane curves. Surface area of surfaces of revolution.
2. Continuity of the function $F(x) = \displaystyle\int_a^x f(t)\ dt, x \in [a, b]$, when $f : [a, b] \longrightarrow \mathbb{R}$ is Riemann integrable. First and Second Fundamental Theorems of Calculus.
3. Mean value theorem. Integration by parts formula. Leibnitz's Rule.
4. Definition of two types of improper integrals. Necessary and sufficient conditions for convergence.
5. Absolute convergence. Comparison and limit comparison tests for convergence.
6. Gamma and Beta functions and their properties. Relationship between them (without proof).

Reference Books

1. Sudhir Ghorpade, Balmohan Limaye; A Course in Calculus and Real Analysis (second edition); Springer.
2. R.R. Goldberg; Methods of Real Analysis; Oxford and IBH Pub. Co., New Delhi, 1970.
3. Calculus and Analytic Geometry (Ninth Edition); Thomas and Finney; Addison-Wesley, Reading Mass., 1998.
4. T. Apostol; Calculus Vol. 2; John Wiley.

Additional Reference Books

1. Ajit Kumar, S.Kumaresan; A Basic Course in Real Analysis; CRC Press, 2014
2. D. Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
3. K. Stewart; Calculus, Booke/Cole Publishing Co, 1994.
4. J. E. Marsden, A.J. Tromba and A. Weinstein; Basic Multivariable Calculus; Springer.
5. R.G. Brtle and D. R. Sherbert; Introduction to Real Analysis Second Ed. ; John Wiley, New Yorm, 1992.

6. M. H. Protter; Basic Elements of Real Analysis; Springer-Verlag, New York, 1998.

USMT/UAMT 302: Linear Algebra I

Unit I. System of Equations, Matrices (15 Lectures)

1. Systems of homogeneous and non-homogeneous linear equations, Simple examples of finding solutions of such systems. Geometric and algebraic understanding of the solutions. Matrices (with real entries), Matrix representation of system of homogeneous and non-homogeneous linear equations. Algebra of solutions of systems of homogeneous linear equations. A system of homogeneous linear equations with number of unknowns more than the number of equations has infinitely many solutions.

2. Elementary row and column operations. Row equivalent matrices. Row reduction (of a matrix to its row echelon form). Gaussian elimination. Applications to solving systems of linear equations. Examples.

3. Elementary matrices. Relation of elementary row operations with elementary matrices. Invertibility of elementary matrices. Consequences such as (i) a square matrix is invertible if and only if its row echelon form is invertible. (ii) invertible matrices are products of elementary matrices. Examples of the computation of the inverse of a matrix using Gauss elimination method.

Unit II. Vector space over $\mathbb{R}$ (15 Lectures)

1. Definition of a vector space over $\mathbb{R}$. Subspaces; criterion for a nonempty subset to be a subspace of a vector space. Examples of vector spaces, including the Euclidean space $\mathbb{R}^n$, lines, planes and hyperplanes in $\mathbb{R}^n$ passing through the origin, space of systems of homogeneous linear equations, space of polynomials, space of various types of matrices, space of real valued functions on a set.

2. Intersections and sums of subspaces. Direct sums of vector spaces. Quotient space of a vector space by its subspace.

3. Linear combination of vectors. Linear span of a subset of a vector space. Definition of a finitely generated vector space. Linear dependence and independence of subsets of a vector space.

4. Basis of a vector space. Basic results that any two bases of a finitely generated vector space have the same number of elements. Dimension of a vector space. Examples. Bases of a vector space as a maximal linearly independent sets and as minimal generating sets.

Unit III. Determinants, Linear Equations (Revisited) (15 Lectures)

1. Inductive definition of the determinant of a $n \times n$ matrix (e. g. in terms of expansion along the first row). Example of a lower triangular matrix. Laplace expansions along an arbitrary row or column. Determinant expansions using permutations
   $\left(\det(A) = \sum_{\sigma \in S_n} \mathrm{sign}(\sigma) \prod_{i=1}^{n} a_{\sigma(i),i}\right)$.

2. Basic properties of determinants (Statements only); (i) $\det A = \det A^T$. (ii) Multilinearity and alternating property for columns and rows. (iii) A square matrix A is invertible if and only if $\det A \neq 0$. (iv) Minors and cofactors. Formula for A^{-1} when $\det A \neq 0$. (v) $\det(AB) = \det A \det B$.

3. Row space and the column space of a matrix as examples of vector space. Notion of row rank and the column rank. Equivalence of the row rank and the column rank. Invariance of rank upon elementary row or column operations. Examples of computing the rank using row reduction.

4. Relation between the solutions of a system of non-homogeneous linear equations and the associated system of homogeneous linear equations. Necessary and sufficient condition for a system of non-homogeneous linear equations to have a solution [viz., the rank of the coefficient matrix equals the rank of the augmented matrix $[A|B]$]. Equivalence of statements (in which A denotes an $n \times n$ matrix) such as the following.

   (i) The system $A\boldsymbol{x} = \boldsymbol{b}$ of non-homogeneous linear equations has a unique solution.

   (ii) The system $A\boldsymbol{x} = \mathbf{0}$ of homogeneous linear equations has no nontrivial solution.

   (iii) A is invertible.

   (iv) $\det A \neq 0$.

   (v) $\operatorname{rank}(A) = n$.

5. Cramers Rule. LU Decomposition. If a square matrix A is a matrix that can be reduced to row echelon form U by Gauss elimination without row interchanges, then A can be factored as $A = LU$ where L is a lower triangular matrix.

Reference books

1 Howard Anton, Chris Rorres, Elementary Linear Algebra, Wiley Student Edition).

2 Serge Lang, Introduction to Linear Algebra, Springer.

3 S Kumaresan, Linear Algebra - A Geometric Approach, PHI Learning.

4 Sheldon Axler, Linear Algebra done right, Springer.

5 Gareth Williams, Linear Algebra with Applications, Jones and Bartlett Publishers.

6 David W. Lewis, Matrix theory.

USMT303: Ordinary Differential Equations

Unit I. Higher order Linear Differential equations (15 Lectures)

1. The general $n-$th order linear differential equations, Linear independence, An existence and uniqueness theorem, the Wronskian, Classification: homogeneous and non-homogeneous, General solution of homogeneous and non-homogeneous LDE, The Differential operator and its properties.

2. Higher order homogeneous linear differential equations with constant coefficients, the auxiliary equations, Roots of the auxiliary equations: real and distinct, real and repeated, complex and complex repeated.

3. Higher order homogeneous linear differential equations with constant coefficients, the method of undermined coefficients, method of variation of parameters.

4. The inverse differential operator and particular integral, Evaluation of $\frac{1}{f(D)}$ for the functions like e^{ax}, $\sin ax$, $\cos ax$, x^m, $x^m \sin ax$, $x^m \cos ax$, $e^{ax}V$ and xV where V is any function of x,

5. Higher order linear differential equations with variable coefficients:
   The Cauchy's equation: $x^3 \frac{d^3y}{dx^3} + x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = f(x)$ and
   The Legendre's equation: $(ax+b)^3 \frac{d^3y}{dx^3} + (ax+b)^2 \frac{d^2y}{dx^2} + (ax+b) \frac{dy}{dx} + y = f(x)$.

Reference Books

1. Units 5, 6, 7 and 8 of E.D. Rainville and P.E. Bedient; Elementary Differential Equations; Macmillan.

2. Units 5, 6 and 7 of M.D. Raisinghania; Ordinary and Partial Differential Equations; S. Chand.

Unit II. Systems of First Order Linear Differential Equations (15 Lectures)

(a) Existence and uniqueness theorem for the solutions of initial value problems for a system of two first order linear differential equations in two unknown functions x, y of a single independent variable t, of the form $\begin{cases} \frac{dx}{dt} = F(t,x,y) \\ \frac{dy}{dt} = G(t,x,y) \end{cases}$ (Statement only).

(b) Homogeneous linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form $\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y, \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y. \end{cases}$.

(c) Wronskian for a homogeneous linear system of first order linear differential equations in two functions x, y of a single independent variable t. Vanishing properties of the Wronskian. Relation with linear independence of solutions.

(d) Homogeneous linear systems with constant coefficients in two unknown functions x, y of a single independent variable t. Auxiliary equation associated to a homogenous system of equations with constant coefficients. Description fo the general solution depending on the roots and their multiplicities of the auxiliary equation, proof of independence of the solutions. Real form of solutions in case the auxiliary equation has complex roots.

(e) Non-homogeneous linear system of linear system of two first order differential equations in two unknown functions of a single independent variable t, of the form
$$\begin{cases} \frac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \\ \frac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t). \end{cases}$$
General Solution of non-homogeneous system. Relation between the solutions of a system

of non-homogeneous linear differential equations and the associated system of homogeneous linear differential equations.

Reference Books

1. G.F. Simmons; Differential Equations with Applications and Historical Notes; Taylor's and Francis.

Unit III. Numerical Solution of Ordinary Differential Equations (15 lectures)

1. Numerical Solution of initial value problem of first order ordinary differential equation using:
   (i) Taylor's series method,
   (ii) Picard's method for successive approximation and its convergence,
   (iii) Euler's method and error estimates for Euler's method,
   (iv) Modified Euler's Method,
   (v) Runge-Kutta method of second order and its error estimates,
   (vi) Runge-Kutta fourth order method.
2. Numerical solution of simultaneous and higher order ordinary differential equation using:
   (i) Runge-Kutta fourth order method for solving simultaneous ordinary differential equation,
   (ii) Finite difference method for the solution of two point linear boundary value problem.

Reference Books

1. Units 8 of S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.

Additional Reference Books

1. E.D. Rainville and P.E. Bedient, Elementary Differential Equations, Macmillan.
2. M.D. Raisinghania, Ordinary and Partial Differential Equations, S. Chand.
3. G.F. Simmons, Differential Equations with Applications and Historical Notes, Taylor's and Francis.
4. S. S. Sastry, Introductory Methods of Numerical Analysis, PHI.
5. K. Atkinson, W.Han and D Stewart, Numerical Solution of Ordinary Differential Equations, Wiley.

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USMT P03 / UAMT P03: Practicals

Suggested Practicals for USMT 301/ UAMT 301

1. Examples of convergent / divergent series and algebra of convergent series.
2. Tests for convergence of series.
3. Calculation of upper sum, lower sum and Riemann integral.
4. Problems on properties of Riemann integral.
5. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.
6. Convergence of improper integrals, different tests for convergence. Beta Gamma Functions.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT302 / UAMT 302

1. Systems of homogeneous and non-homogeneous linear equations.
2. Elementary row/column operations and Elementary matrices.
3. Vector spaces, Subspaces.
4. Linear Dependence/independence, Basis, Dimension.
5. Determinant and Rank of a matrix.
6. Solution to a system of linear equations, LU decomposition
7. Miscellaneous Theory Questions.
8. Miscellaneous theory questions from units I, II and III.

Suggested Practicals For USMT 303

1. Finding the general solution of homogeneous and non-homogeneous higher order linear differential equations.
2. Solving higher order linear differential equations using method of undetermined coefficients and method of variation of parameters.
3. Solving a system of first order linear ODES have auxiliary equations with real and complex roots.
4. Finding the numerical solution of initial value problems using Taylor's series method, Picard's method, modified Euler's method, Runge-Kutta method of fourth order and calculating their accuracy.
5. Finding the numerical solution of simultaneous ordinary differential equation using fourth order Runge-Kutta method.
6. Finding the numerical solution of two point linear boundary value problem using Finite difference method.

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Semester-IV

Note: Unless indicated otherwise, proofs of the results mentioned in the syllabus should be covered.

USMT 401/ UAMT 401: Multivariable Calculus I

UNIT I. Functions of Several Variables (15 Lectures)

1. Review of vectors in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and basic notions such as addition and scalar multiplication, inner product, length (norm), and distance between two points.

2. Real-valued functions of several variables (Scalar fields). Graph of a function. Level sets (level curves, level surfaces, etc). Examples. Vector valued functions of several variables (Vector fields). Component functions. Examples.

3. Sequences, Limits and Continuity: Sequence in $\mathbb{R}^n$ [with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$] and their limits. Neighbourhoods in $\mathbb{R}^n$. Limits and continuity of scalar fields. Composition of continuous functions. Sequential characterizations. Algebra of limits and continuity (Results with proofs). Iterated limits.
   Limits and continuity of vector fields. Algebra of limits and continuity vector fields. (without proofs).

4. Partial and Directional Derivatives of scalar fields: Definitions of partial derivative and directional derivative of scalar fields (with emphasis on $\mathbb{R}^2$ and $\mathbb{R}^3$). Mean Value Theorem of scalar fields.

UNIT II. Differentiation of Scalar Fields (15 Lectures)

1. Differentiability of scalar fields (in terms of linear transformation). The concept of (total) derivative. Uniqueness of total derivative of a differentiable function at a point. Examples of functions of two or three variables. Increment Theorem. Basic properties including (i) continuity at a point of differentiability, (ii)existence of partial derivatives at a point of differentiability, and (iii) differentiability when the partial derivatives exist and are continuous.

2. Gradient. Relation between total derivative and gradient of a function. Chain rule. Geometric properties of gradient. Tangent planes.

3. Euler's Theorem.

4. Higher order partial derivatives. Mixed Partial Theorem (n=2).

UNIT III. Applications of Differentiation of Scalar Fields and Differentiation of Vector Fields (15 lectures)

1. Applications of Differentiation of Scalar Fields: The maximum and minimum rate of change of scalar fields. Taylor's Theorem for twice continuously differentiable functions. Notions of local maxima, local minima and saddle points. First Derivative Test. Examples. Hessian matrix. Second Derivative Test for functions of two variables. Examples. Method of Lagrange Multipliers.

2. Differentiation of Vector Fields: Differentiability and the notion of (total) derivative. Differentiability of a vector field implies continuity, Jacobian matrix. Relationship between total derivative and Jacobian matrix. The chain rule for derivative of vector fields (statements only).

Reference books

1. T. Apostol; Calculus, Vol. 2 (Second Edition); John Wiley.
2. Sudhir Ghorpade, Balmohan Limaye; A Course in Multivariable Calculus and Analysis (Second Edition); Springer.
3. Walter Rudin; Principles of Mathematical Analysis; McGraw-Hill, Inc.
4. J. E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus; Springer.
5. D.Somasundaram and B.Choudhary; A First Course in Mathematical Analysis, Narosa, New Delhi, 1996.
6. K. Stewart; Calculus; Booke/Cole Publishing Co, 1994.

Additional Reference Books

1. Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, (Ninth Edition); Addison-Wesley, 1998.
2. Howard Anton; Calculus- A new Horizon,(Sixth Edition); John Wiley and Sons Inc, 1999.
3. S L Gupta and Nisha Rani; Principles of Real Analysis; Vikas Publishing house PVT LTD.
4. Shabanov, Sergei; Concepts in Calculus, III: Multivariable Calculus; University Press of Florida, 2012.
5. S C Malik and Savita Arora; Mathematical Analysis; New Age International Publishers.

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USMT402/UAMT402: Linear Algebra II

UNIT I. Linear Transformations

1. Definition of a linear transformation of vector spaces; elementary properties. Examples. Sums and scalar multiples of linear transformations. Composites of linear transformations. A Linear transformation of $V \longrightarrow W$, where V, W are vector spaces over $\mathbb{R}$ and V is a finite-dimensional vector space is completely determined by its action on an ordered basis of V.

2. Null-space (kernel) and the image (range) of a linear transformation. Nullity and rank of a linear transformation. Rank-Nullity Theorem (Fundamental Theorem of Homomorphisms).

3. Matrix associated with linear transformation of $V \longrightarrow W$ where V and W are finite dimensional vector spaces over $\mathbb{R}$.. Matrix of the composite of two linear transformations. Invertible linear transformations (isomorphisms), Linear operator, Effect of change of bases on matrices of linear operator.

4. Equivalence of the rank of a matrix and the rank of the associated linear transformation. Similar matrices.

UNIT II. Inner Products and Orthogonality

1. Inner product spaces (over $\mathbb{R}$). Examples, including the Euclidean space $\mathbb{R}^n$ and the space of real valued continuous functions on a closed and bounded interval. Norm associated to an inner product. Cauchy-Schwarz inequality. Triangle inequality.

2. Angle between two vectors. Orthogonality of vectors. Pythagoras theorem and some geometric applications in $\mathbb{R}^2$. Orthogonal sets, Orthonormal sets. Gram-Schmidt orthogonalizaton process. Orthogonal basis and orthonormal basis for a finite-dimensional inner product space.

3. Orthogonal complement of any set of vectors in an inner product space. Orthogonal complement of a set is a vector subspace of the inner product space. Orthogonal decomposition of an inner product space with respect to its subspace. Orthogonal projection of a vector onto a line (one dimensional subspace). Orthogonal projection of an inner product space onto its subspace.

UNIT III. Eigenvalues, Eigenvectors and Diagonalisation

1. Eigenvalues and eigenvectors of a linear transformation of a vector space into itself and of square matrices. The eigenvectors corresponding to distinct eigenvalues of a linear transformation are linearly independent. Eigen spaces. Algebraic and geometric multiplicity of an eigenvalue.

2. Characteristic polynomial. Properties of characteristic polynomials (only statements). Examples. Cayley-Hamilton Theorem. Applications.

3. Invariance of the characteristic polynomial and eigenvalues of similar matrices.

4. Diagonalisable matrix. A real square matrix A is diagonalisable if and only if there is a basis of $\mathbb{R}^n$ consisting of eigenvectors of A. (Statement only - $A_{n\times n}$ is diagonalisable if and only if sum of algebraic multiplicities is equal to sum of geometric multiplicities of all the eigenvalues of $A \ = n$). Procedure for diagonalising a matrix.

5. Spectral Theorem for Real Symmetric Matrices (Statement only). Examples of orthogonal diagonalisation of real symmetric matrices. Applications to quadratic forms and classification of conic sections.

Reference books

1. Howard Anton, Chris Rorres; Elementary Linear Algebra; Wiley Student Edition).

2. Serge Lang; Introduction to Linear Algebra; Springer.

3. S Kumaresan; Linear Algebra - A Geometric Approach; PHI Learning.

4. Sheldon Axler; Linear Algebra done right; Springer.

5. Gareth Williams; Linear Algebra with Applications; Jones and Bartlett Publishers.

6. David W. Lewis; Matrix theory.

USMT403A: Numerical Methods (Elective A)

Unit I. Solution of Algebraic and Transcendental Equations (15L)

1. Measures of Errors: Relative, absolute and percentage errors, Accuracy and precision: Accuracy to n decimal places, accuracy to n significant digits or significant figures, Rounding and Chopping of a number, Types of Errors: Inherent error, Round-off error and Truncation error.

2. Iteration methods based on first degree equation: Newton-Raphson method. Secant method. Regula-Falsi method.
   Derivations and geometrical interpretation and rate of convergence of all above methods to be covered.

3. General Iteration method: Fixed point iteration method.

Unit II. Interpolation, Curve fitting, Numerical Integration(15L)

1. Interpolation: Lagrange's Interpolation. Finite difference operators: Forward Difference operator, Backward Difference operator. Shift operator. Newton's forward difference interpolation formula. Newton's backward difference interpolation formula.
   Derivations of all above methods to be covered.

2. Curve fitting: linear curve fitting. Quadratic curve fitting.

3. Numerical Integration: Trapezoidal Rule. Simpson's 1/3 rd Rule. Simpson's 3/8th Rule. Derivations all the above three rules to be covered.

Unit III. Solution Linear Systems of Equations, Eigenvalue problems(15L)

1. Linear Systems of Equations: LU Decomposition Method (Dolittle's Method and Crout's Method). Gauss-Seidel Iterative method.

2. Eigenvalue problems: Jacobi's method for symmetric matrices. Rutishauser method for arbitrary matrices.

Reference Books:

1. Kendall E. and Atkinson; An Introduction to Numerical Analysis; Wiley.

2. M. K. Jain, S. R. K. Iyengar and R. K. Jain; Numerical Methods for Scientific and Engineering Computation; New Age International Publications.

3. S. Sastry; Introductory methods of Numerical Analysis; PHI Learning.

4. An introduction to Scilab-Cse iitb.

Additional Reference Books

1. S.D. Comte and Carl de Boor; Elementary Numerical Analysis, An algorithmic approach; McGrawHillll International Book Company.

2. Hildebrand F.B.; Introduction to Numerical Analysis; Dover Publication, NY.

3. Scarborough James B.; Numerical Mathematical Analysis; Oxford University Press, New Delhi.

USMT403B Statistical Methods and their Applications (Elective B)

Unit I. Descriptive Statistics and random variables (15 Lectures)
Measures of location (mean, median, mode), Partition values and their graphical locations, measures of dispersion, skewness and kurtosis, Exploratory Data Analysis (Five number summary, Box Plot, Outliers), Random Variables (discrete and continuous), Expectation and variance of a random variable.

Unit II. Probability Distributions and Correlation (15 Lectures)
Discrete Probability Distribution (Binomial, Poisson), Continuous Probability Distribution: (Uniform, Normal), Correlation, Karl Pearson's Coefficient of Correlation, Concept of linear Regression, Fitting of a straight line and curve to the given data by the method of least squares, relation between correlation coefficient and regression coefficients.

Unit III. Inferential Statistics (15 lectures)
Population and sample, parameter and statistic, sampling distribution of Sample mean and Sample Variance, concept of statistical hypothesis, critical region, level of significance, confidence interval and two types of errors, Tests of significance (t-test, Z-test, F-test, Chi-Square Test (only applications))

Reference Books

1. Fundamentals of Mathematical Statistics,12th Edition, S. C. Gupta and V. K. Kapoor,Sultan Chand & Sons, 2020.

2. Statistics for Business and Economics, 11th Edition, David R. Anderson, Dennis J. Sweeney and Thomas A. Williams, Cengage Learning, 2011.

3. Introductory Statistics, 8th Edition, Prem S. Mann, John Wiley & Sons Inc., 2013.

4. A First Course in Statistics, 12th Edition, James McClave and Terry Sincich, Pearson Education Limited, 2018.

5. Introductory Statistics, Barbara Illowsky, Susan Dean and Laurel Chiappetta, OpenStax, 2013.

6. Hands-On Programming with R, Garrett Grolemund, O'Reilly.

USMT P04 / UAMT P04: Practicals

Suggested Practical for USMT 401/ UAMT 401

1. Limits and continuity of scalar fields and vector fields, using "definition and otherwise" , iterated limits.
2. Computing directional derivatives, partial derivatives and mean value theorem of scalar fields.
3. Differentiability of scalar field,Total derivative, gradient, level sets and tangent planes.
4. Chain rule, higher order derivatives and mixed partial derivatives of scalar fields.
5. Maximum and minimum rate of change of scalar fields. Taylor's Theorem. Finding Hessian/Jacobean matrix. Differentiation of a vector field at a point. Chain Rule for vector fields.
6. Finding maxima, minima and saddle points. Second derivative test for extrema of functions of two variables and method of Lagrange multipliers.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT402/UAMT 402

1. Linear transformation, Kernel, Rank-Nullity Theorem.
2. Linear Isomorphism, Matrix associated with Linear transformations.
3. Inner product and properties, Projection, Orthogonal complements.
4. Orthogonal, orthonormal sets, Gram-Schmidt orthogonalisation
5. Eigenvalues, Eigenvectors, Characteristic polynomial. Applications of Cayley Hamilton Theorem.
6. Diagonalisation of matrix, orthogonal diagonalisation of symmetric matrix and application to quadratic form.
7. Miscellaneous Theoretical Questions based on full paper.

Suggested Practicals for USMT403A

The Practical no. 1 to 6 should be performed either using non-programable scientific calculators or by using the software Scilab.

1. Newton-Raphson method, Secant method.
2. Regula-Falsi method, Iteration Method..
3. Interpolating polynomial by Lagrange's Interpolation, Newton forward and backward difference Interpolation.
4. Curve fitting, Trapezoidal Rule, Simpson's 1/3rd Rule, Simpson's 3/8th Rule.
5. LU decomposition method, Gauss-Seidel Interative method.

6. Jacobi's method, Rutishauser method..
7. Miscellaneous theoretical questions from all units.

Suggested Practicals for USMT403B
All practicals should be performed using any one of the following softwares: MS Excel, R, Strata, SPSS, Sage Math to carry out data analysis and computations.

1. Descriptive Statistics.
2. Random Variables.
3. Probability Distributions.
4. Correlation and Regression.
5. Testing of hypothesis.
6. Case studies.
7. Miscellaneous Theory questions based on Unit I,II,III.

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Scheme of Examination (75:25)

The performance of the learners shall be evaluated into two parts.

- Internal Assessment of 25 percent marks.
- Semester End Examinations of 75 percent marks.

I. **Internal Evaluation of 25 Marks:**

S.Y.B.Sc. :

(i) One class Test of 20 marks to be conducted during Practical session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).
Q2: Multiple choice 5 questions. (10 Marks: 5×2)
Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Active participation in routine class: 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

S.Y.B.A. :

(i) One class Test of 20 marks to be conducted during Tutorial session.
Paper pattern of the Test:
Q1: Definitions/ Fill in the blanks/ True or False with Justification (04 Marks).

Q2: Multiple choice 5 questions. (10 Marks: 5×2)

Q3: Attempt any 2 from 3 descriptive questions. (06 marks: 2×3)

(ii) Journal : 05 Marks.

OR

Students who are willing to explore topics related to syllabus, dealing with applications historical development or some interesting theorems and their applications can be encouraged to submit a project for 25 marks under the guidance of teachers.

II. **Semester End Theory Examinations :** There will be a Semester-end external Theory examination of 75 marks for each of the courses USMT301/UAMT301, USMT/USAT 302, USMT 303 of Semester III and USMT/UAMT401, USMT/UAMT 402, USMT 403 of semester IV to be conducted by the college.

1. Duration: The examinations shall be of 2 and $\frac{1}{2}$ hours duration.
2. Theory Question Paper Pattern:
   a) There shall be FOUR questions. The first three questions Q1, Q2, Q3 shall be of 20 marks, each based on the units I, II, III respectively. The question Q4 shall be of 15 marks based on the entire syllabus.
   b) All the questions shall be compulsory. The questions Q1, Q2, Q3, Q4 shall have internal choices within the questions. Including the choices, the marks for each question shall be 25-27.
   c) The questions Q1, Q2, Q3, Q4 may be subdivided into sub-questions as a, b, c, d & e, etc and the allocation of marks depends on the weightage of the topic.

III. **Semester End Examinations Practicals:**

At the end of the Semesters III & IV Practical examinations of three hours duration and 150 marks shall be conducted for the courses USMTP03, USMTP04.

At the end of the Semesters III & IV Practical examinations of two hours duration and 100 marks shall be conducted for the courses UAMTP03, UAMTP04.
In semester III, the Practical examinations for USMT301/UAMT301, USMT302/UAMT302 and USMT303 are held together by the college.
In Semester IV, the Practical examinations for USMT401/UAMT401, USMT402/UAMT402 and USMT403 are held together by the college.

Paper pattern: The question paper shall have two parts A and B.
Each part shall have two Sections.

Section I Objective in nature: Attempt any Eight out of Twelve multiple choice questions (04 objective questions from each unit) ($8 \times 3 = 24$ Marks).

Section II Problems: Attempt any Two out of Three (01 descriptive question from each unit) ($8 \times 2 = 16$ Marks).

Practical Course	Part A	Part B	Part C	Marks out of	duration
USMTP03	Questions from USMT301	Questions from USMT302	Questions from USMT 303	120	3 hours
UAMTP03	Questions from UAMT301	Questions from UAMT302	——	80	2 hours
USMTP04	Questions from USMT401	Questions from USMT402	Questions from USMT403	120	3 hours
UAMTP04	Questions from UAMT401	Questions from UAMT402	——	80	2 hours

Marks for Journals and Viva:
For each course USMT301/UAMT301, USMT302/UAMT302, USMT303, USMT401/UAMT401, USMT402/UAMT402, USMT3031:

1. Journal: 10 marks (5 marks for each journal).
2. Viva: 10 marks.

Each Practical of every course of Semester III and IV shall contain 10 (ten) problems out of which minimum 05 (five) have to be written in the journal. .
A student must have a certified journal before appearing for the practical examination.
In case a student does not posses a certified journal he/she will be evaluated for 120/80 marks. He/she is not qualified for Journal + Viva marks.

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